Ge 212 Problem Set 1

Due Monday October 2, 2017

How hard should you work on this problem set? Well, this is a 9-unit class with 6 hours per week of homework time, so the target is 6 hours. Don’t leave it all for the last day.

1. *Exact differentials*

Decide whether each of the following differential expressions is an exact differential, and find an integrating factor for the inexact ones (i.e. if *M*d*x* + *N*d*y* is not exact, there exists a function (*x*,*y*) such that (*M*/)d*x* + (*N*/)d*y* is an exact differential; the trick is to find it):

* 1. (*R*/*P*)d*T* – (*RT*/*P*2)d*P* (*R* is a constant)
  2. *xy*d*x* + *xy*d*y*
  3. (*x*/*y*2)d*x* – (*x*/*y*3)d*y*
  4. *xy*2d*x – x2y*d*y* (fun bonus: find a general formula for **all** functions (*x*,*y*) that work for this problem)

1. *Fun with partial derivative transformations and Jacobians*

Here are two different definitions of the thermodynamic Grüneisen parameter:

and .

Prove that these definitions are equivalent, i.e. 1 = 2. I claim that the easiest way to do this is, after expanding ∂lnX = (1/X)∂X, to break both partial derivatives into ratios of Jacobians, and separately deal with the numerators and denominators. You may or may not need four additional fun facts: (∂*X*)Y = –(∂*Y*)X, d*E* = *T*d*S* – *P*d*V*, the equality of mixed 2nd partials of an exact differential, and the cyclic rule (See the table of Jacobians <http://www.asimow.com/jacobiantable.jpg> and note that *U* and *E* are the same thing).

1. *Legendre transformation*

Consider a two variable function *a*(*b*, *c*). Its total differential is written in the usual way d*a* = *p* d*b + q* d*c*, where *p* ≡ (∂*a*/∂*b*)c and *q* ≡ (∂*a*/∂*c*)b. The two-variable Legendre transform of *a* is defined by *f* = *f*(*p*, *q*) ≡ *a* – *pb* – *qc*.

   i) Show that, for **any** function *a(b,c)*, the total differential of its two-variable Legendre transform is just d*f* = *–b* d*p – c* d*q*.

ii) Now consider a specific function, *a(b,c)* = *b*3 – 2*c*2. Find its two-variable Legendre transform *f*(*p,q*) and demonstrate that the transform is correct [i.e., show that (∂*f*/∂*p*)q = –*b* and (∂*f*/∂*q*)p = –*c,* such thatthe reverse transform would give back *a = f + bp + cq*].