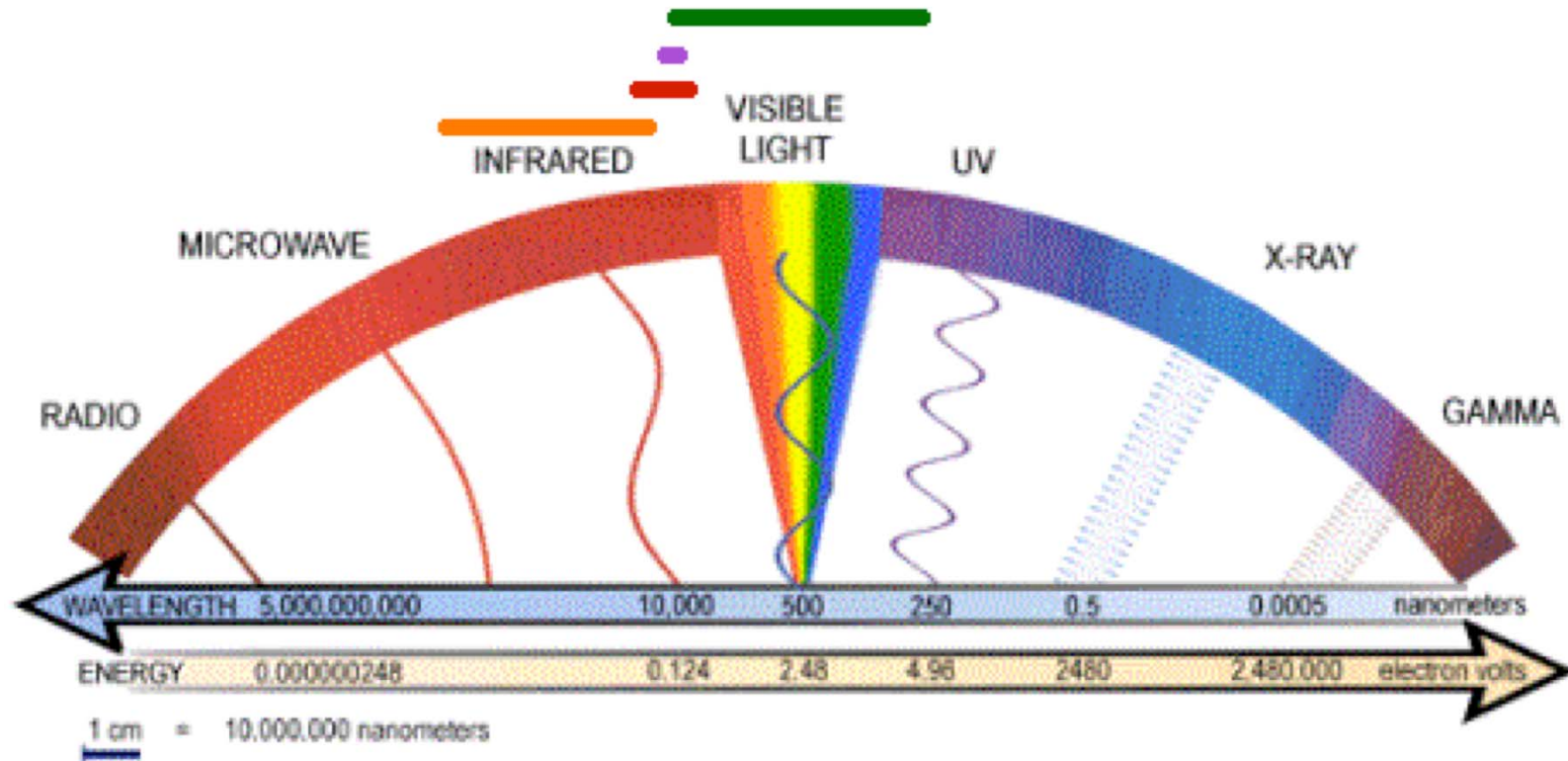


X-ray Powder Diffraction, Part 2: Review & Applications

February 22, 2010

Jennifer Jackson

Electromagnetic spectrum



X-rays: $\lambda \sim 0.5 \text{ nm}$

less than and/or comparable to interatomic positions

Ideal for diffracting conditions for atoms \rightarrow crystal structure

Determining crystal structure & volume

- Determines *d-spacing* & *atomic positions*
- 3D crystal structure
- **Mineral ID!**
- Provides volume, hence density if chemistry is known (EMPA+SEM)
- Xtal Structure dictates behavior
 - Solid solution
 - $(\text{Mg,Fe})_2\text{SiO}_4$ olivine
 - Vibrational modes
 - Raman, IR spectra
 - Ion-exchange
 - Zeolites
- Structural transitions



calcite

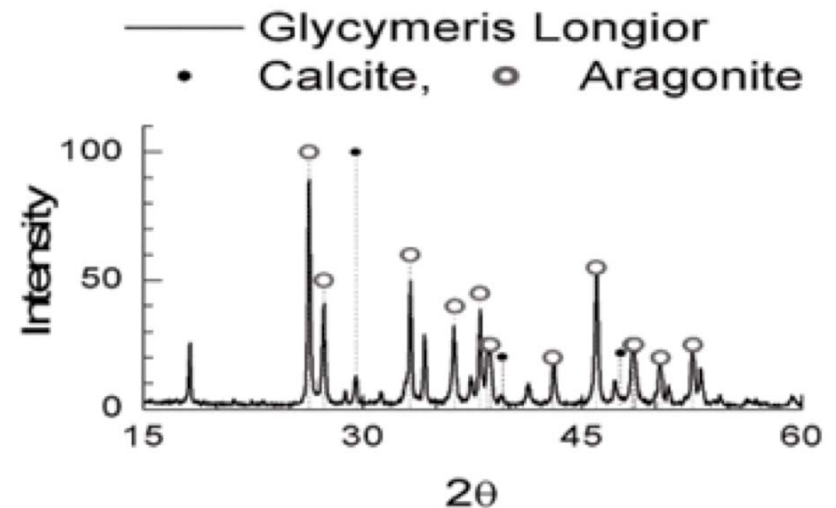
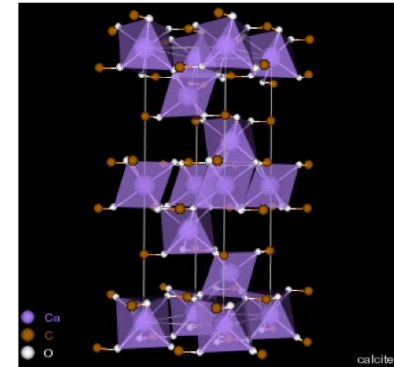


FIG. 1: XRD spectra of fossil shells, species Glycymeris Longior. Scattered points show data for Aragonite (open circles) and Calcite.

Specific information from x-ray diffraction

- Intensity of the diffracted photons, I , contains all static information of a crystal:

- scattering factor, f
 - Atomic number, scattering angle, atomic thermal motion (Debye-Waller factor)
- Structure factor, F
 - Summations of the scattering factors

- Atomic spacing

- Atomic positions
 - From relative intensities from the same material
- Structural information (space group): mineral ID
- Lattice parameters
- Volume
- Density (if chemistry is known)

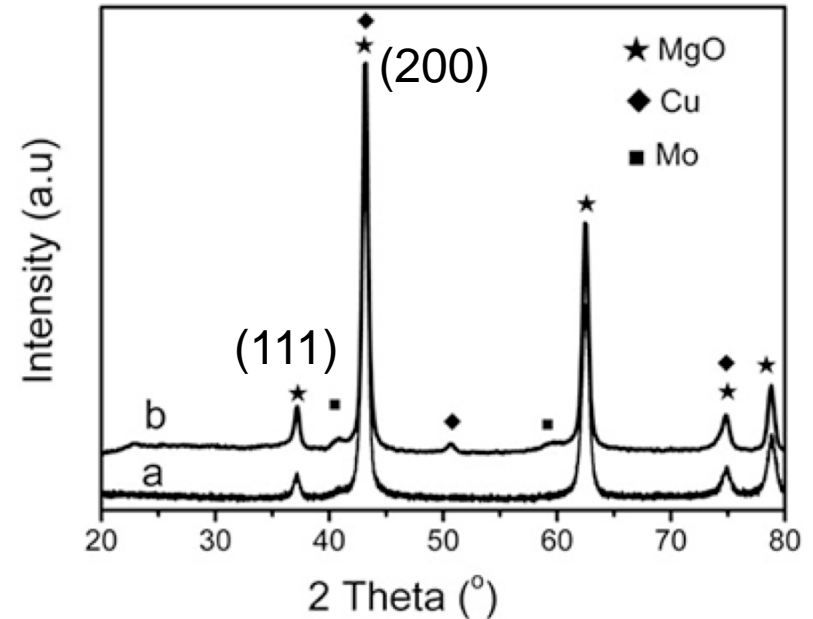
$$I = k \left[\sum_j f_j e^{i2\pi(hx_j + ky_j + lz_j)} \right]^2 = kF_{hkl}^2$$

Other useful information from the spectrum:

- Phase proportions
 - Using relative intensities of the different phases present
- Strain analysis
 - Using peak widths
- Volume change with controlled thermodynamic constraints (P - T)
- Identify polymorphs (olivine and high-pressure phases, like wadsleyite)
- Structural Phase transitions
 - Identify polymorphs with certainty
 - In-situ monitoring of phase transitions
 - Determine the pressure (depth) at a fixed temperature, of a phase transition, for example.

Determining volume

- Cubic symmetry (NaCl):
 $d^2 = a^2(h^2+k^2+l^2)$
 $n\lambda = 2d\sin\theta$
- Important to fit all the peaks using weighted fits
 - Increased resolution at higher angles
- e.g., Celref: Fits peak positions, assuming space group & atomic positions
- If chemistry is known, one can accurately determine the density,
 - $\rho = \text{mass/volume}$
 - Watch unit conversions



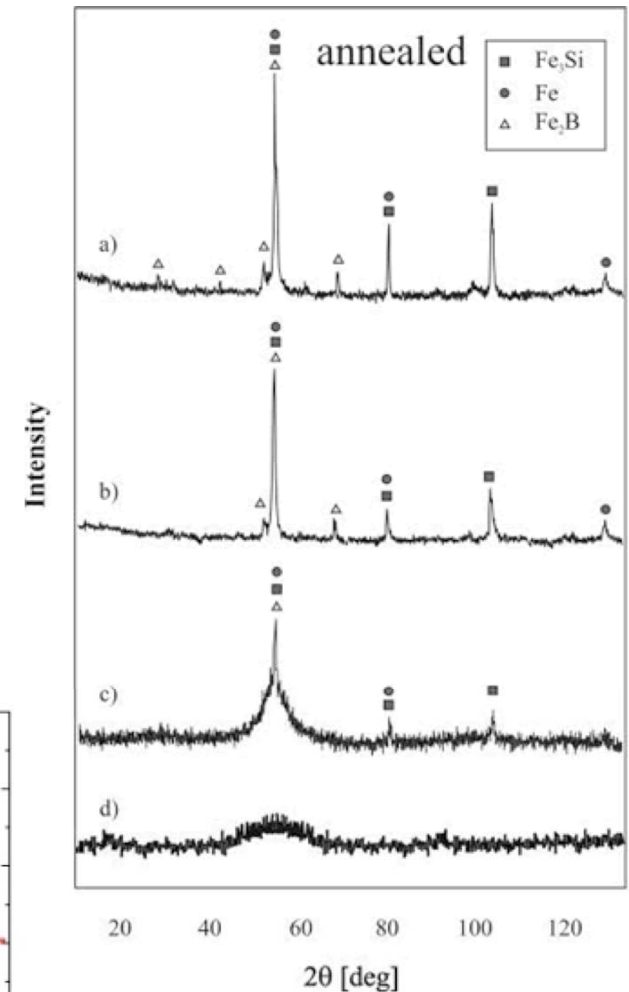
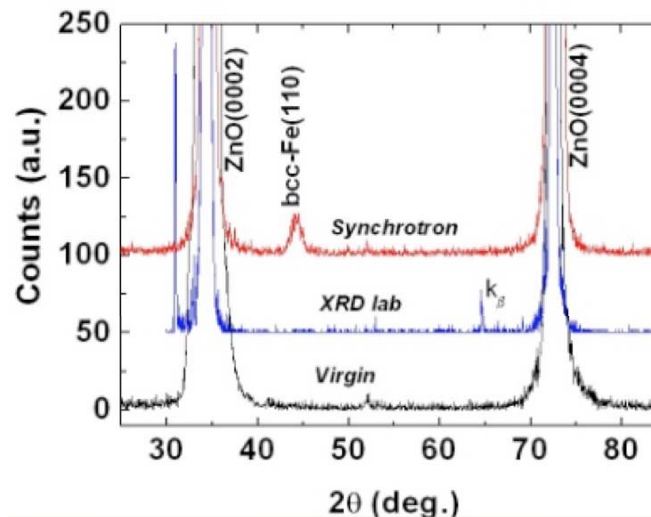
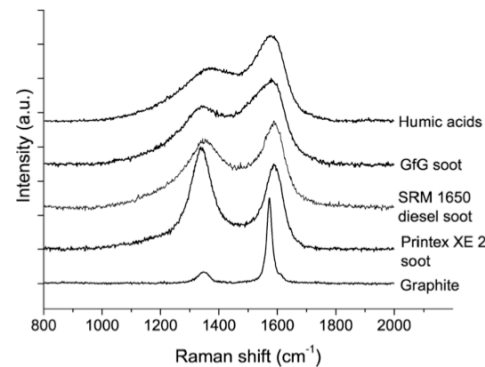
Origin of peak widths

Instrument

- Source wavelength
- Detector resolution

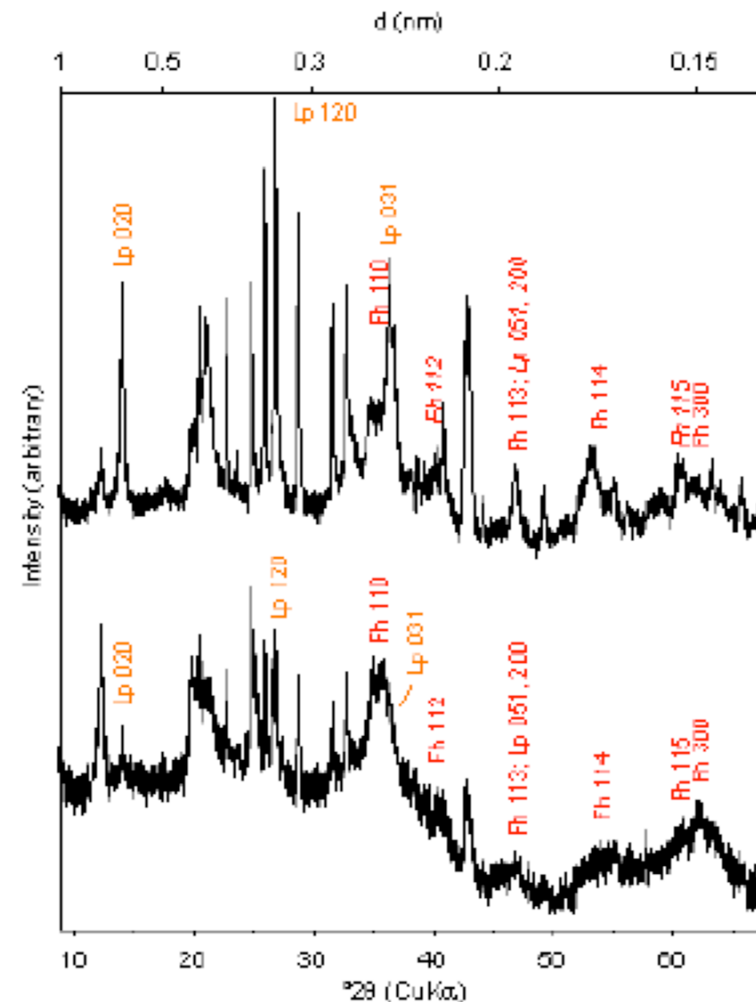
Sample

- Adsorption:
 - OH, organics
- Disorder
 - Mg-Fe, Ca-Na
- Strain
 - Tempered steel
- Grain size
 - If grain size is $\sim \lambda$, get broadening
- Temperature
 - Debye-waller factor
 - Thermal ellipsoids
 - Exists at 300 K



Mixed phases: determining proportion

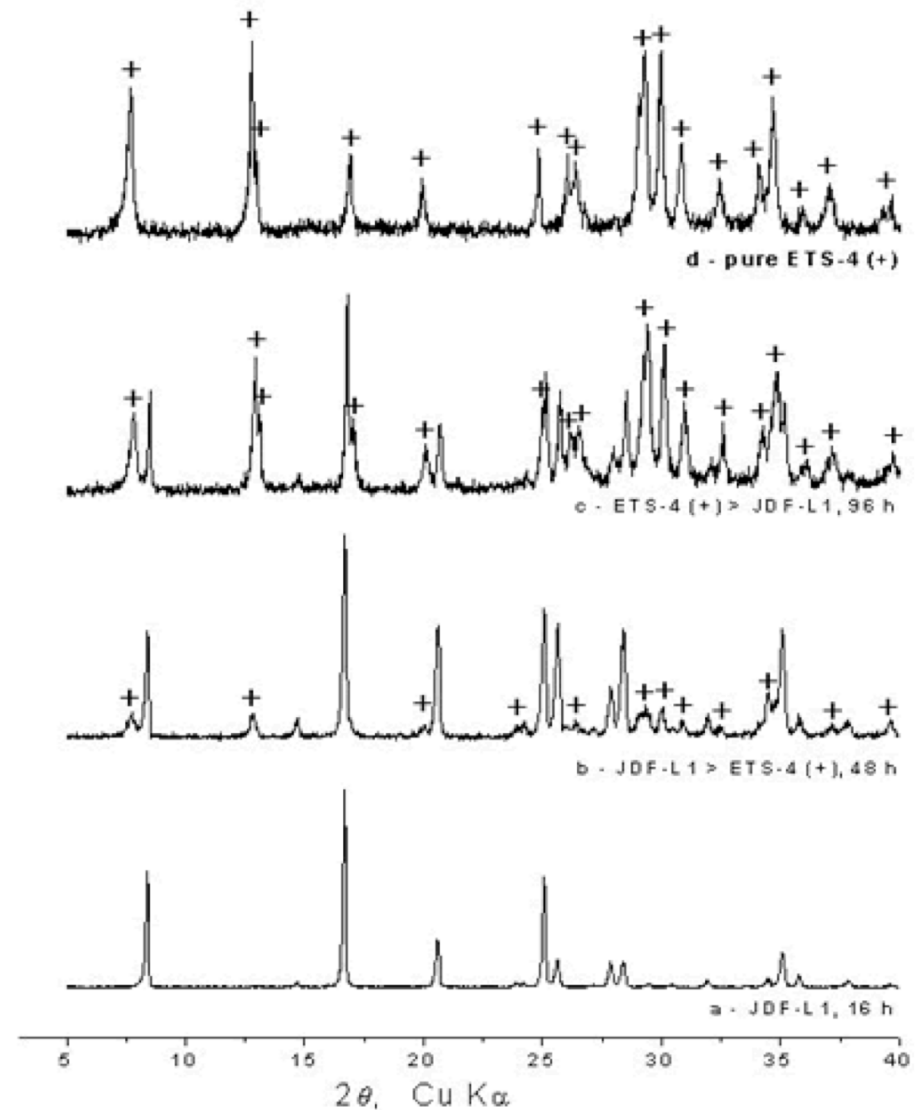
- Environmental sciences
- Quantity and type of clay is important
- Acid mine drainage areas
- Requires an analysis of relative intensities
 - Watch out for preferred orientation



Two iron oxide coatings on lignite from Sokolov showing different relative proportions of kaolinite (not marked), ferrihydrite and lepidocrocite.

Determining phase transitions

- See evolution of diffraction peaks as a function of temperature, time, pressure, hydration, etc.
- Example: hydrothermal studies on titanosilicates:
 $\text{Na}_4\text{Ti}_2\text{Si}_8\text{O}_{22} \cdot 4\text{H}_2\text{O}$ to
 $\text{Na}_9\text{Si}_{12}\text{Ti}_5\text{O}_{38}(\text{OH}) \cdot 12\text{H}_2\text{O}$



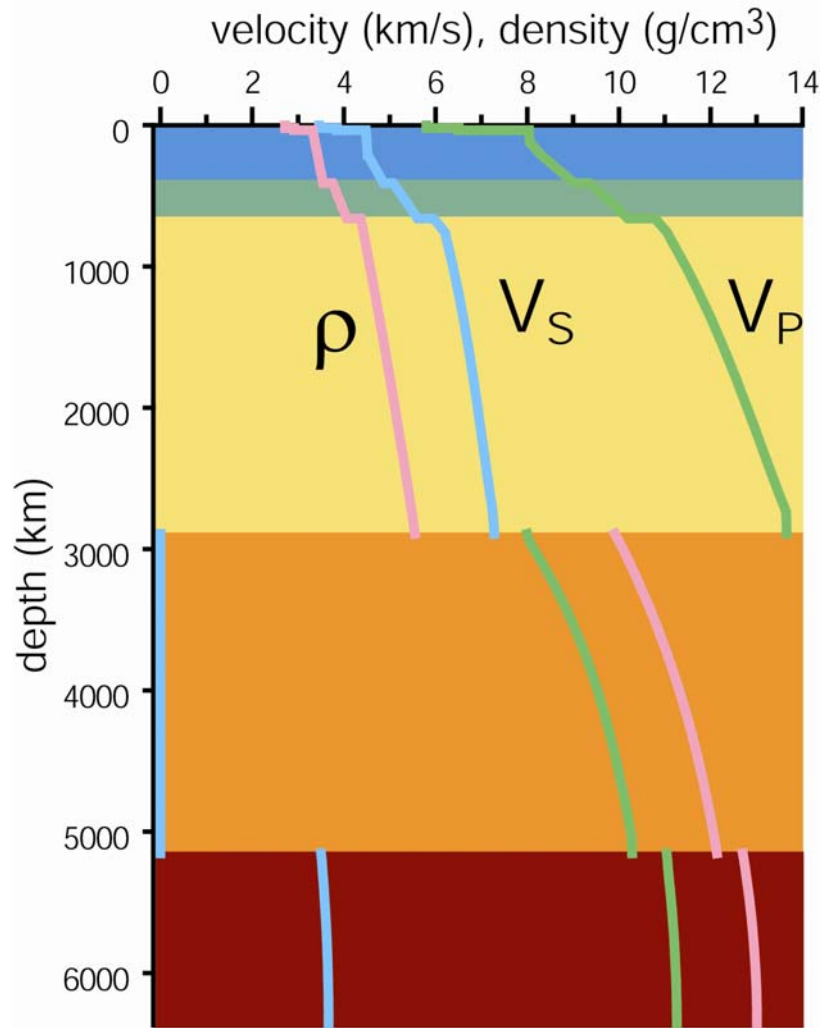
Determining the crystal structure and density of minerals in the interior of Earth

Preliminary Reference Earth Model (PREM)

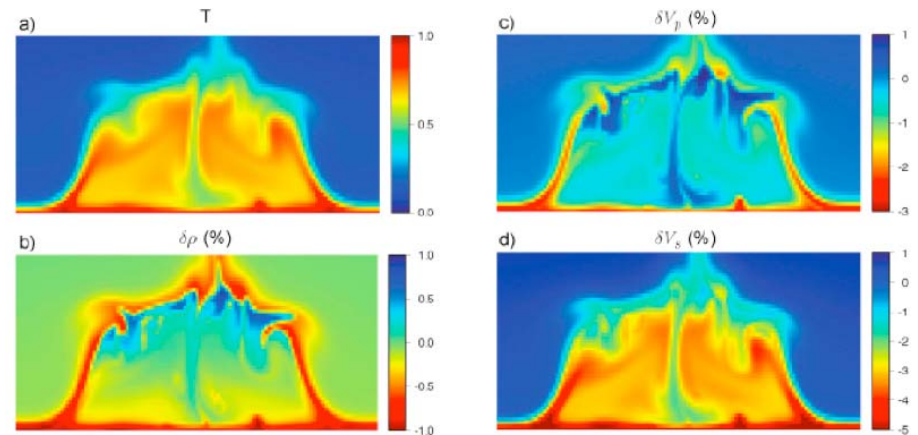
*Measure candidate minerals
At high-PT conditions:*

V_p and V_s

Density (XRD + chemical info)

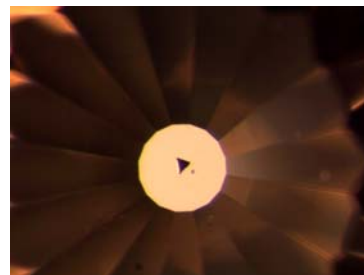
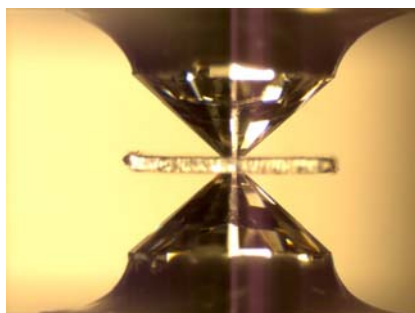
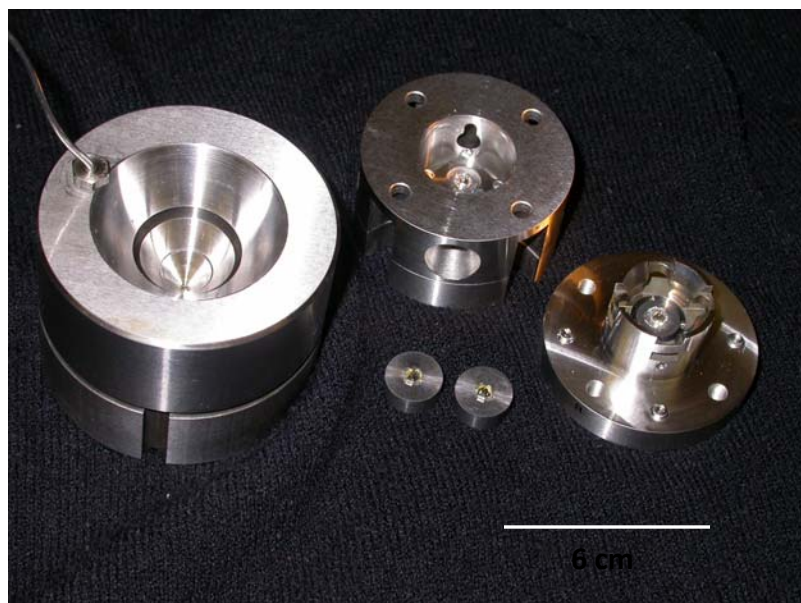


(Dziewonski & Anderson 1981)

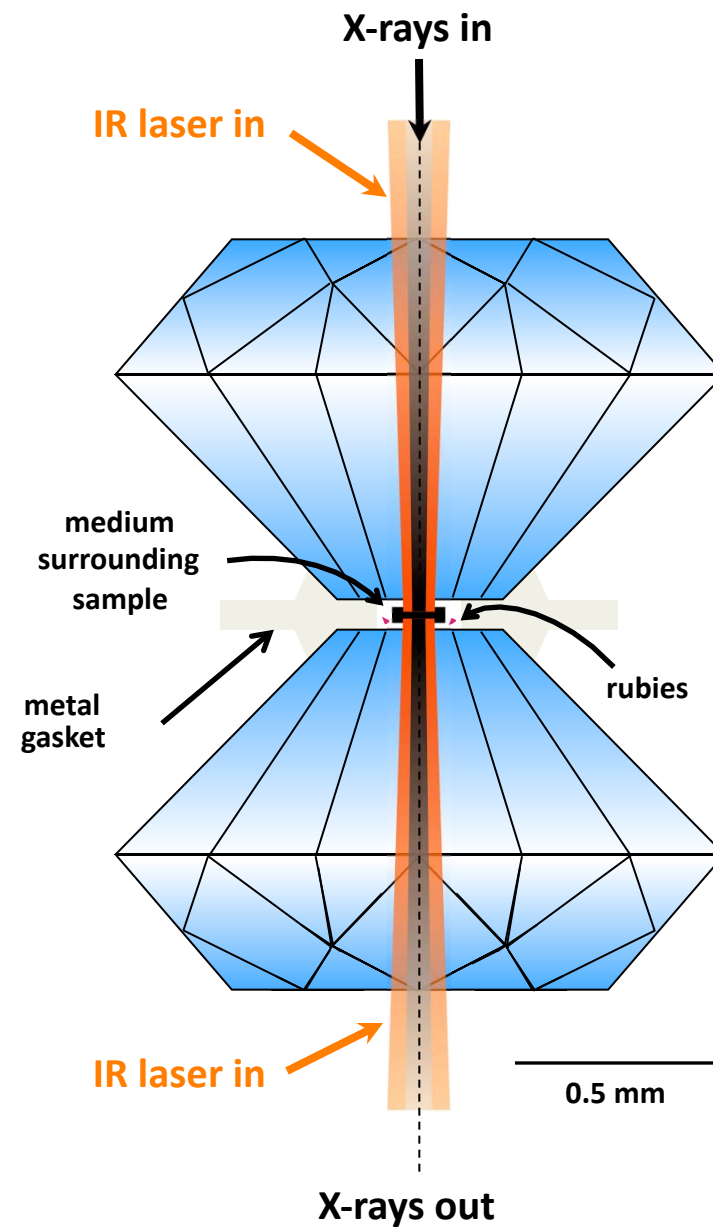


(Tan & Gurnis 2005)

The diamond-anvil cell



Diamond-anvil-cell laboratory,
Caltech



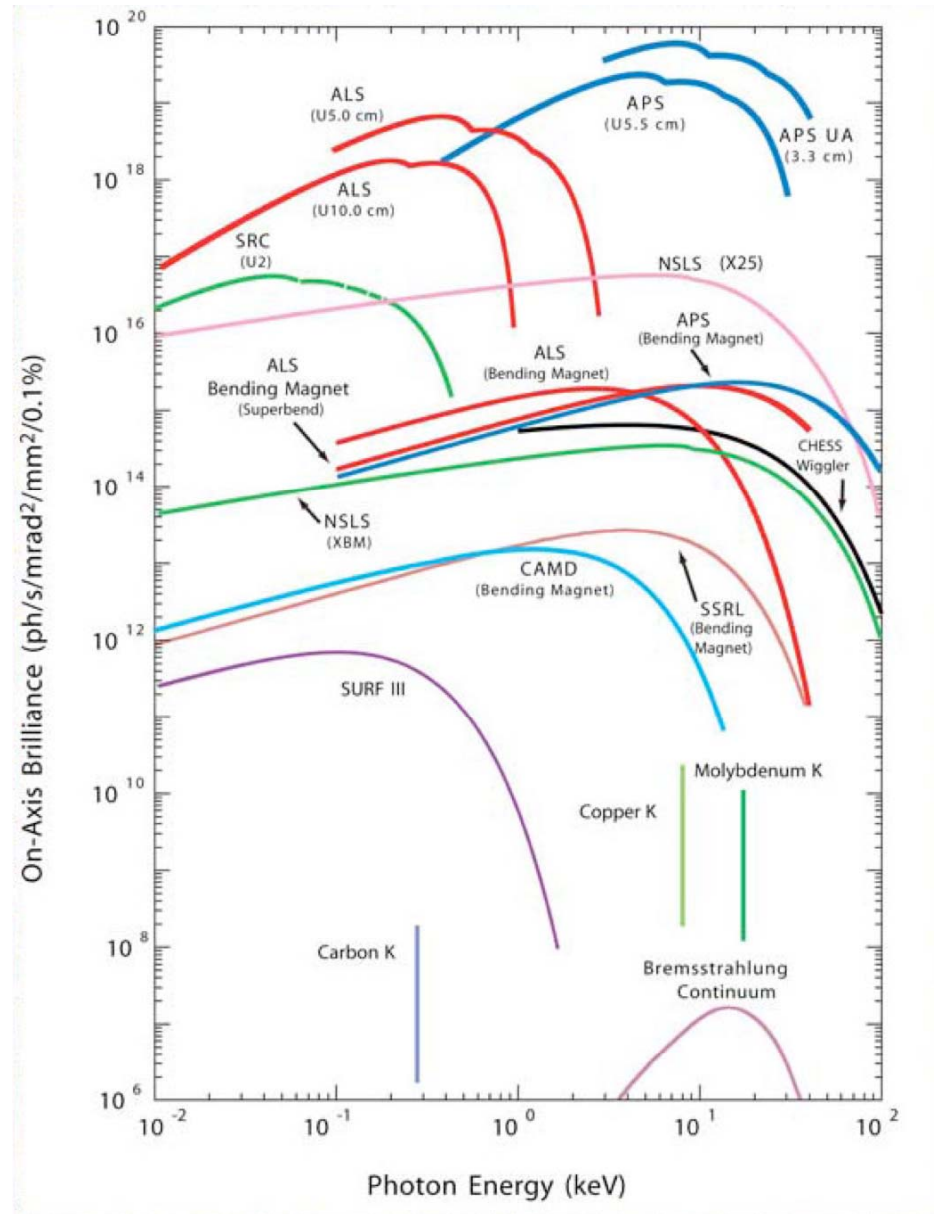
Synchrotron x-radiation



Advanced Photon Source (APS)
Argonne National Laboratory, IL



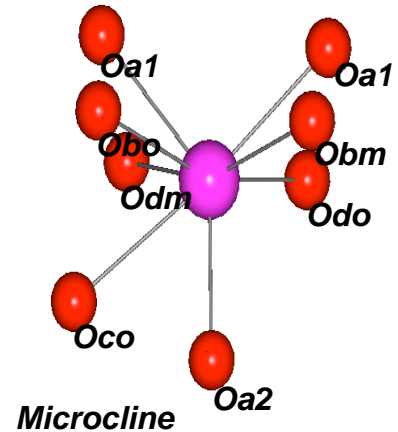
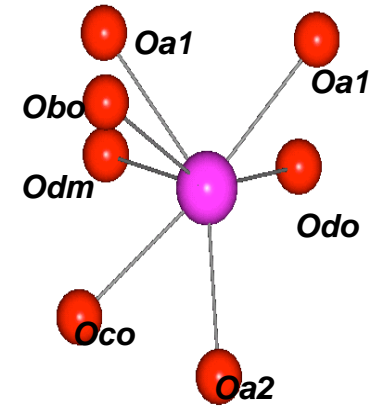
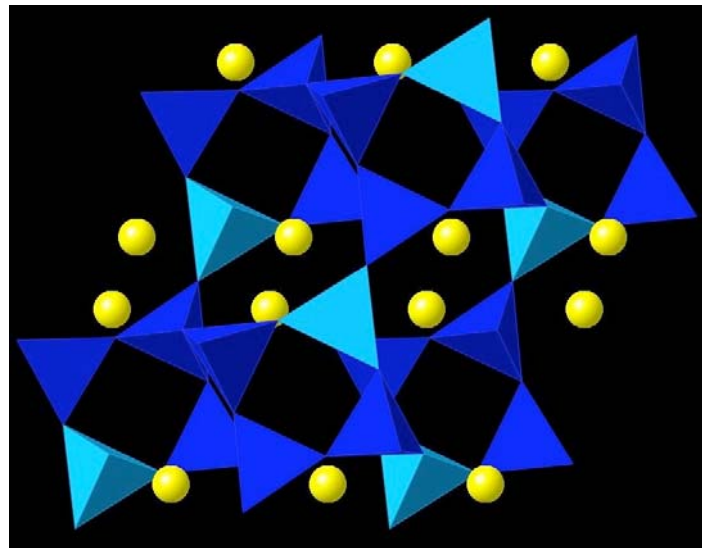
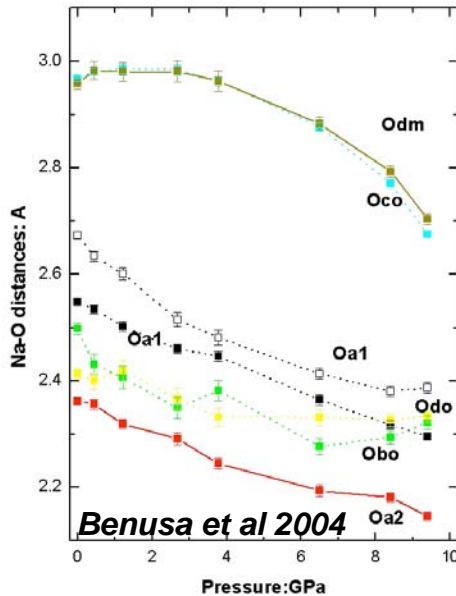
Storage ring: electromagnets



Crystal structures at high-pressures

As framework minerals compress, the polyhedra compress and tilt, and the extra-framework cavity changes shape

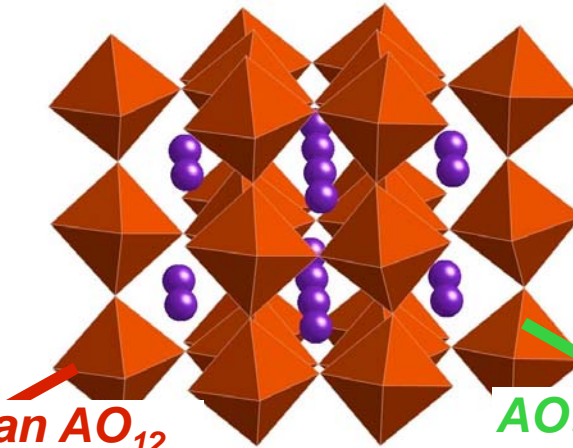
- Single crystal microcline, KAlSi_3O_8
- Cation partitioning
- Retention & Diffusion of non-bonded species
 - *Ar, K, Xe*
- Elastic and hence, thermodynamic properties



Allan & Angel (1997) EJM

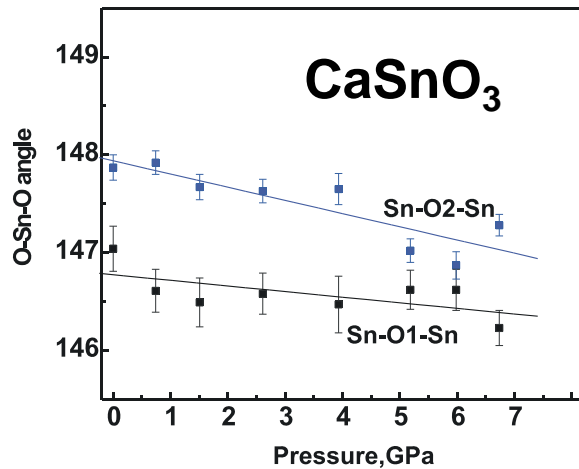
More structures: ABO_3 perovskites

As BO_6 rotate, the A-O bonds are compressed

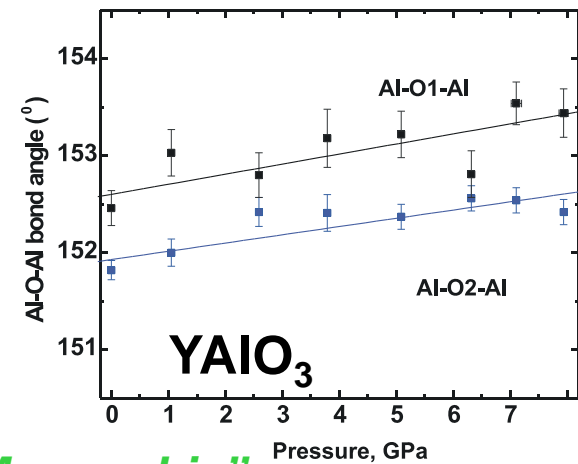


BO_6 stiffer than AO_{12}

AO_{12} stiffer than BO_6



“Less cubic”

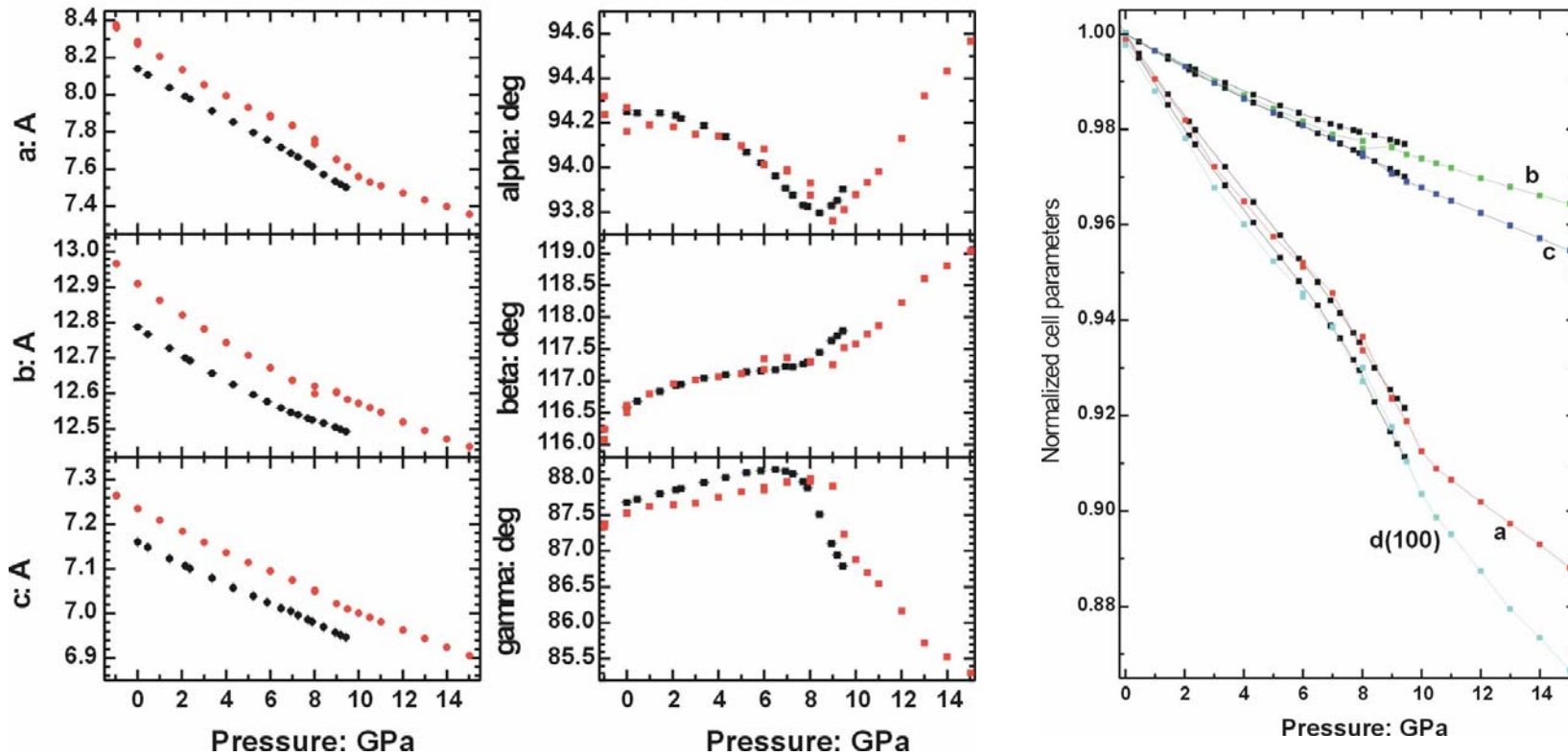


“More cubic”

(Zhao et al. 2005)

Albite ($\text{NaAlSi}_3\text{O}_8$) response to pressure

- Unit cell parameters provide more information
 - Experiments went up to 10 GPa, supplemented with density functional theory (DFT) calculations
 - 60% of albite compression is along (100)
 - Cell angles show softening



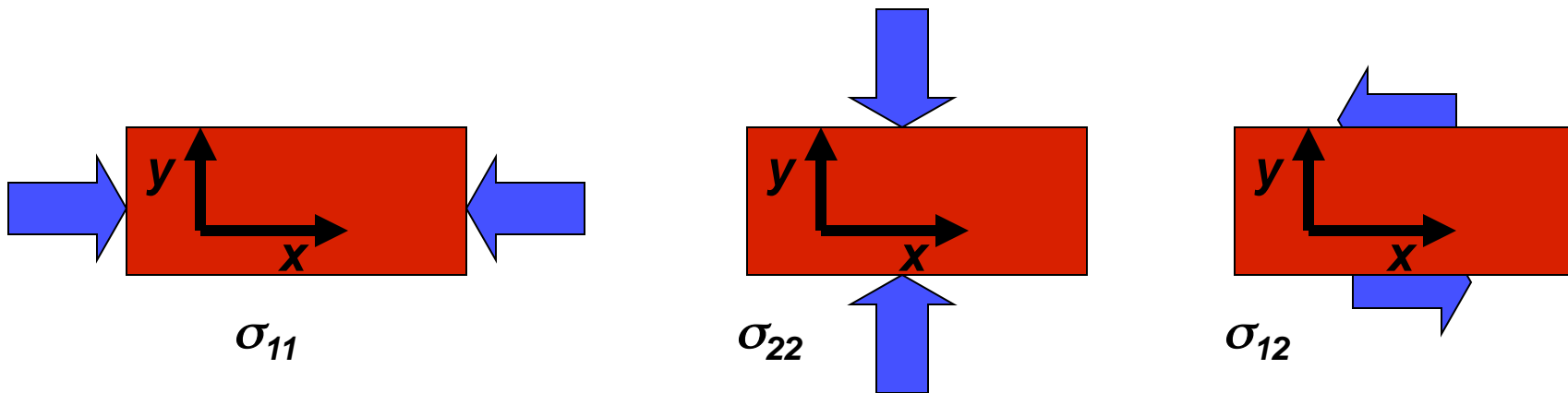
Elastic tensor

□ Elastic Compliance Tensor (s_{ijkl}):

- 4th rank tensor defining a physical quantity relating stress to the resulting strain within the linear elastic regime.

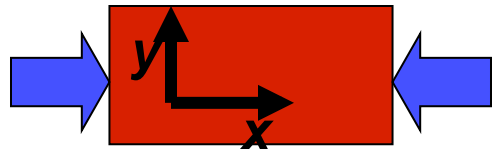
Generalized Hooke's Law:

$$\varepsilon_{ij} = s_{ijkl} \sigma_{kl}$$



Compliance = softness Modulus = stiffness

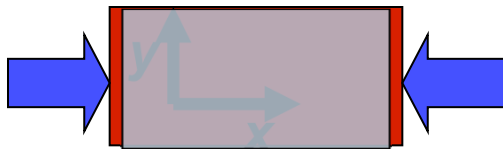
Strain and compliance tensors



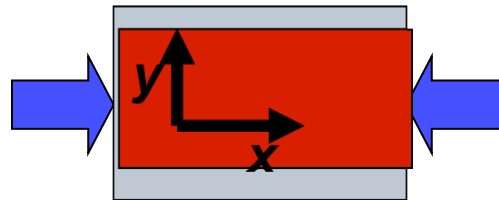
$$\sigma_{11}$$

Generalized Hooke's Law:

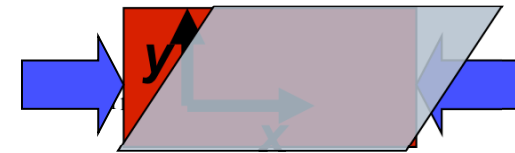
$$\varepsilon_{ij} = s_{ijkl} \sigma_{kl}$$



$$\varepsilon_{11} = s_{1111} \sigma_{11}$$



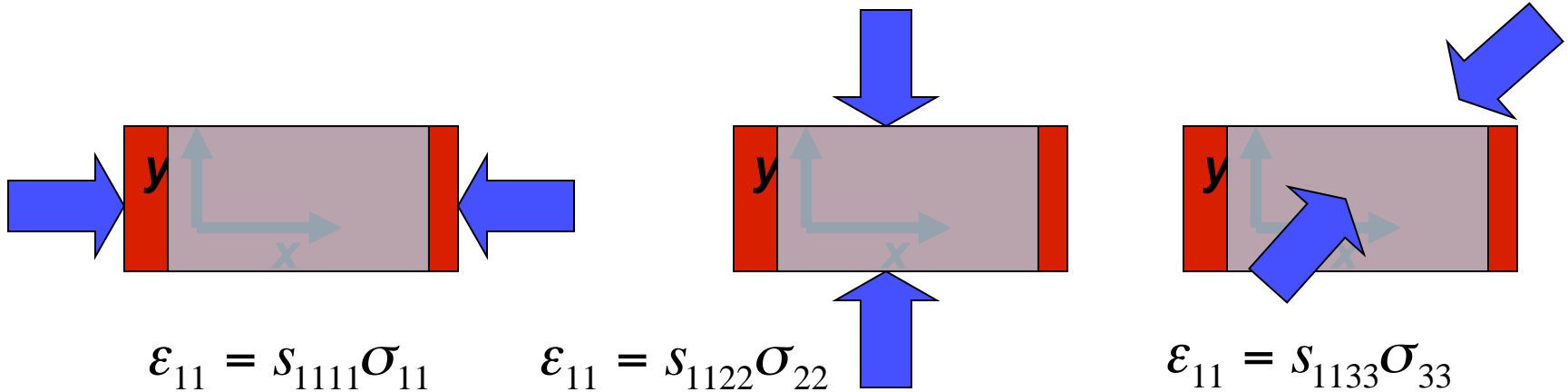
$$\varepsilon_{22} = s_{2211} \sigma_{11}$$



$$\varepsilon_{12} = s_{1211} \sigma_{11}$$

- Potentially 81 components of s_{ijkl}
- Stress and strain must be symmetric
 - $\sigma_{ij} = \sigma_{ji}$ and $\varepsilon_{ij} = \varepsilon_{ji}$
 - s_{ijkl} must be symmetric in itself
 -21 independent components of s_{ijkl}
 - Less if symmetry higher than triclinic

Hydrostatic experiments



□ No shear stress

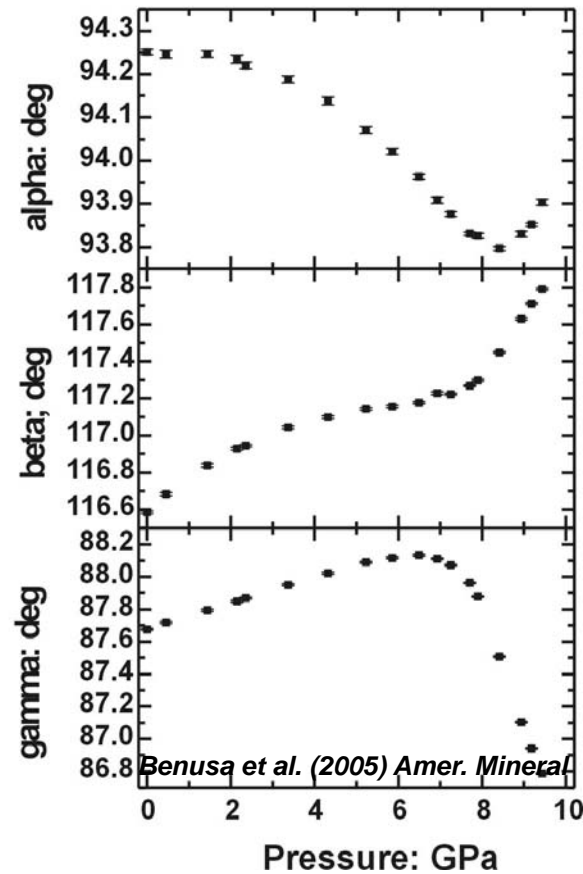
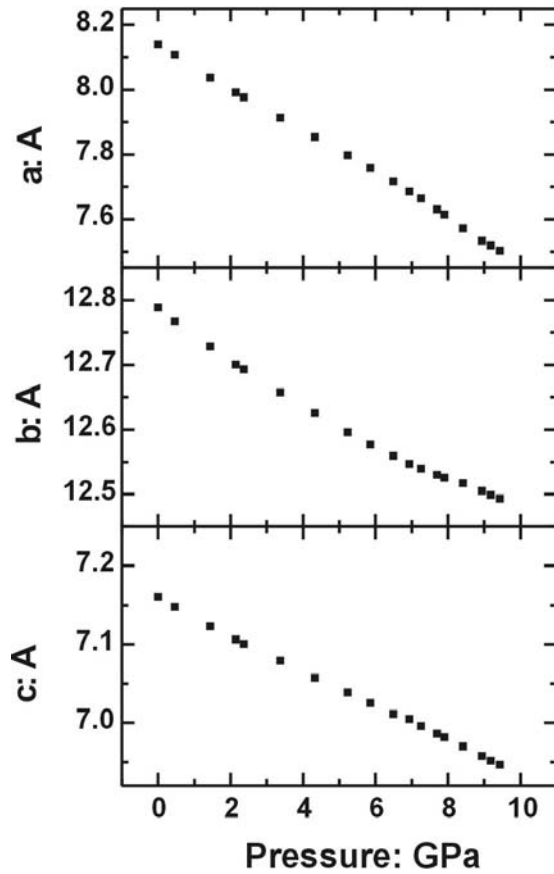
- $\sigma_{ij} = 0$
- $\sigma_{11} = \sigma_{22} = \sigma_{33} = -P$

$$\begin{aligned}
 \epsilon_{11} &= s_{1111}\sigma_{11} + s_{1122}\sigma_{22} + s_{1133}\sigma_{33} \\
 &= -(s_{1111} + s_{1122} + s_{1133})P \\
 -\epsilon_{11}/P &= (s_{1111} + s_{1122} + s_{1133})
 \end{aligned}$$

□ Changes in cell parameters are strains

- Six components for triclinic crystals

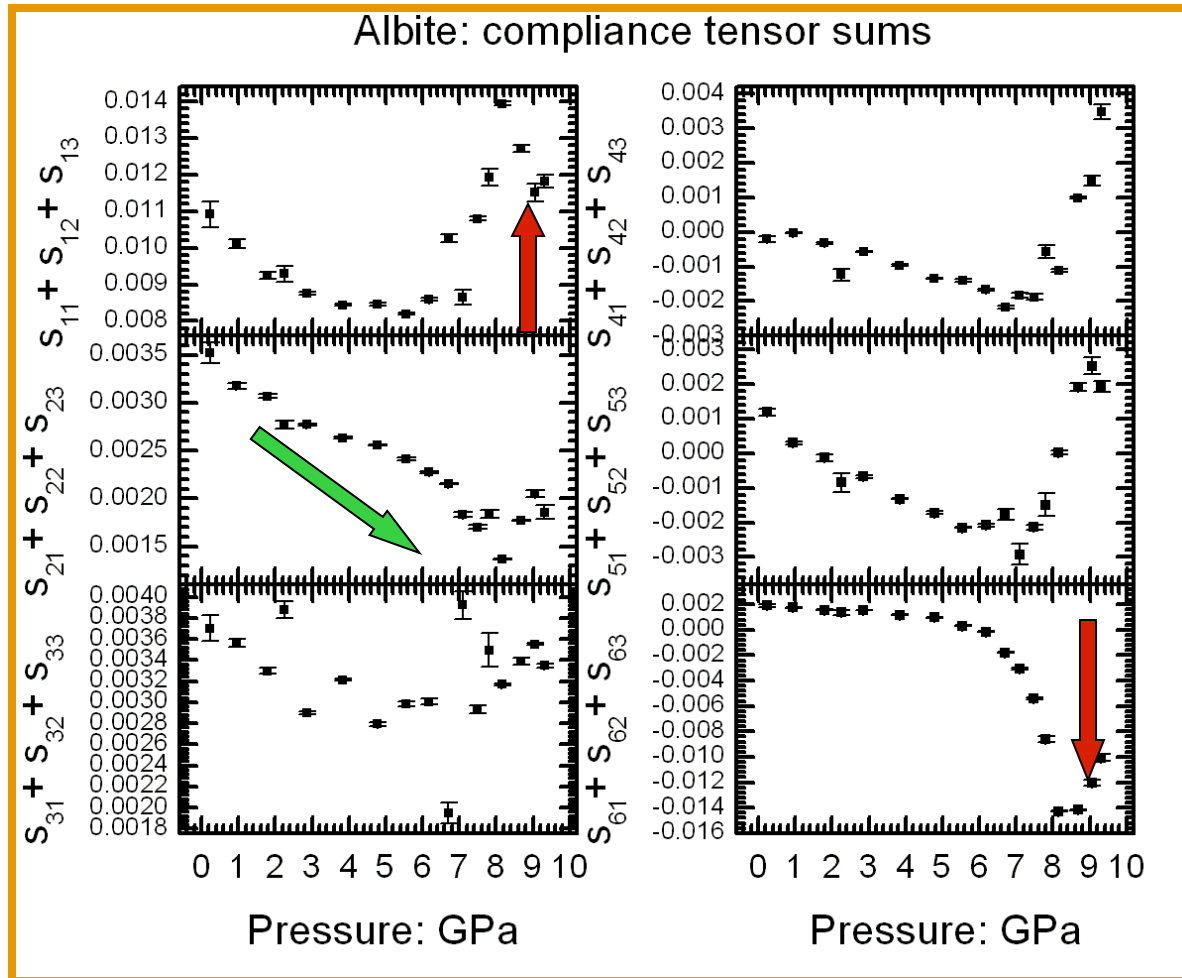
Obtaining compressional moduli



□ Triclinic cell parameters

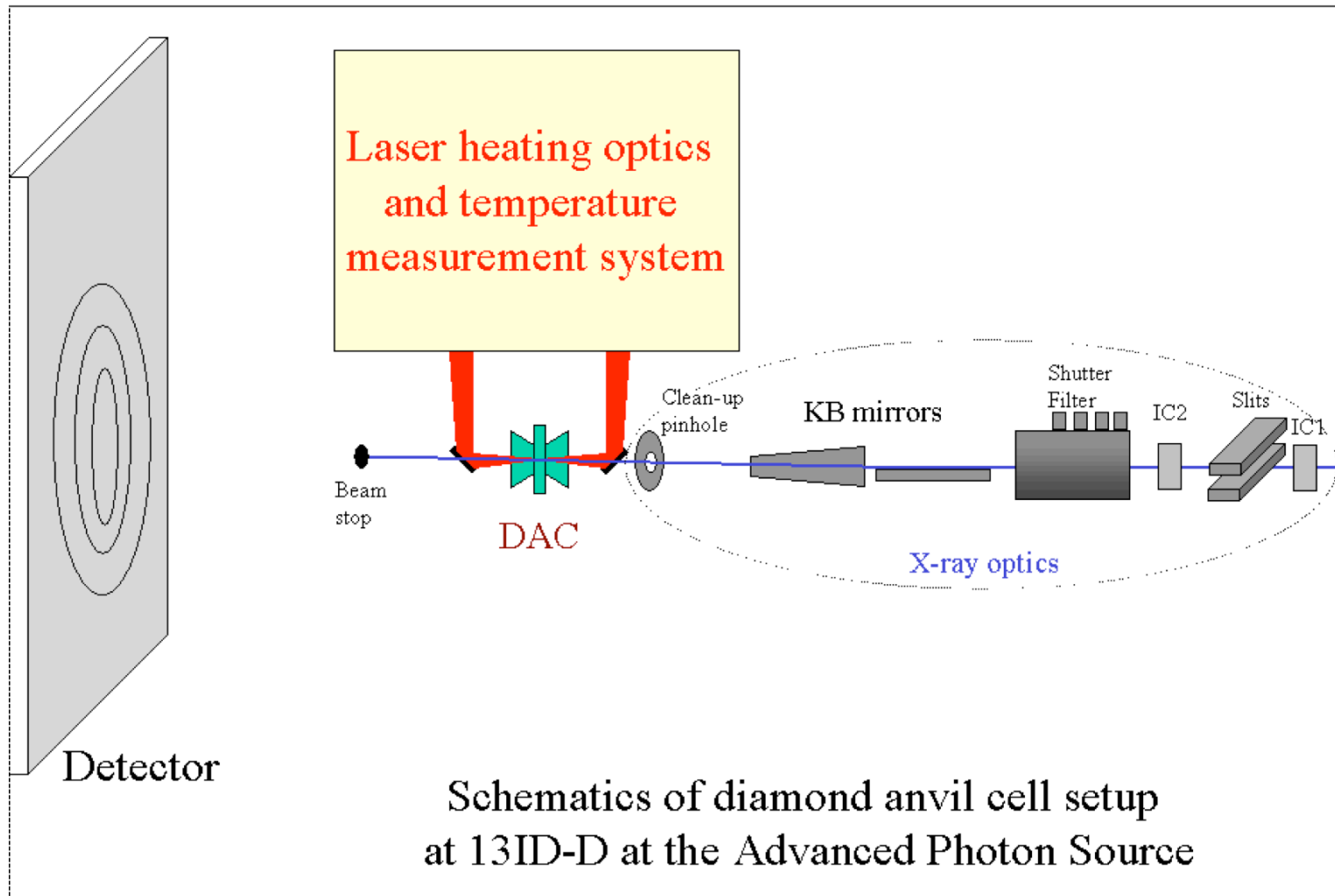
Benusa et al. (2005) Amer. Mineral

Obtaining compressional moduli



- Triclinic cell parameters
- Calculate incremental strain tensor
- Calculate s_{1j} at pressure
- Fit to obtain s_{1j} at room pressure

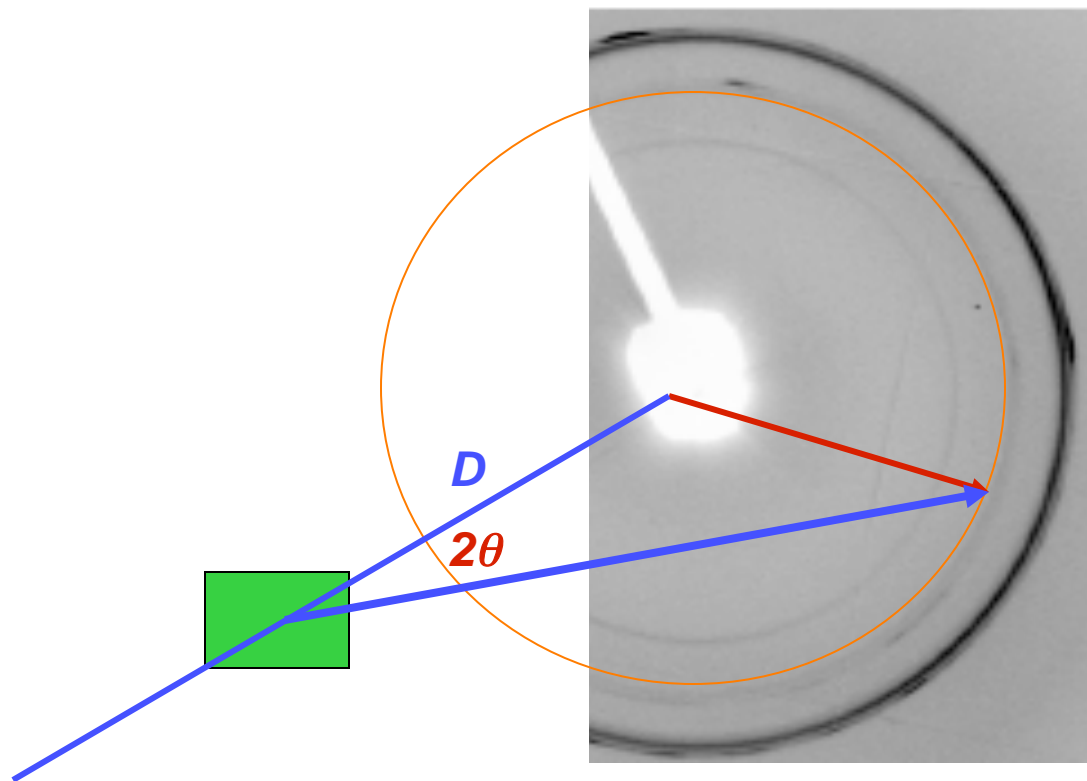
Synchrotron powder x-ray diffraction



$$\lambda = 0.04133 \text{ nm} = 30 \text{ keV}$$

Synchrotron powder x-ray diffraction

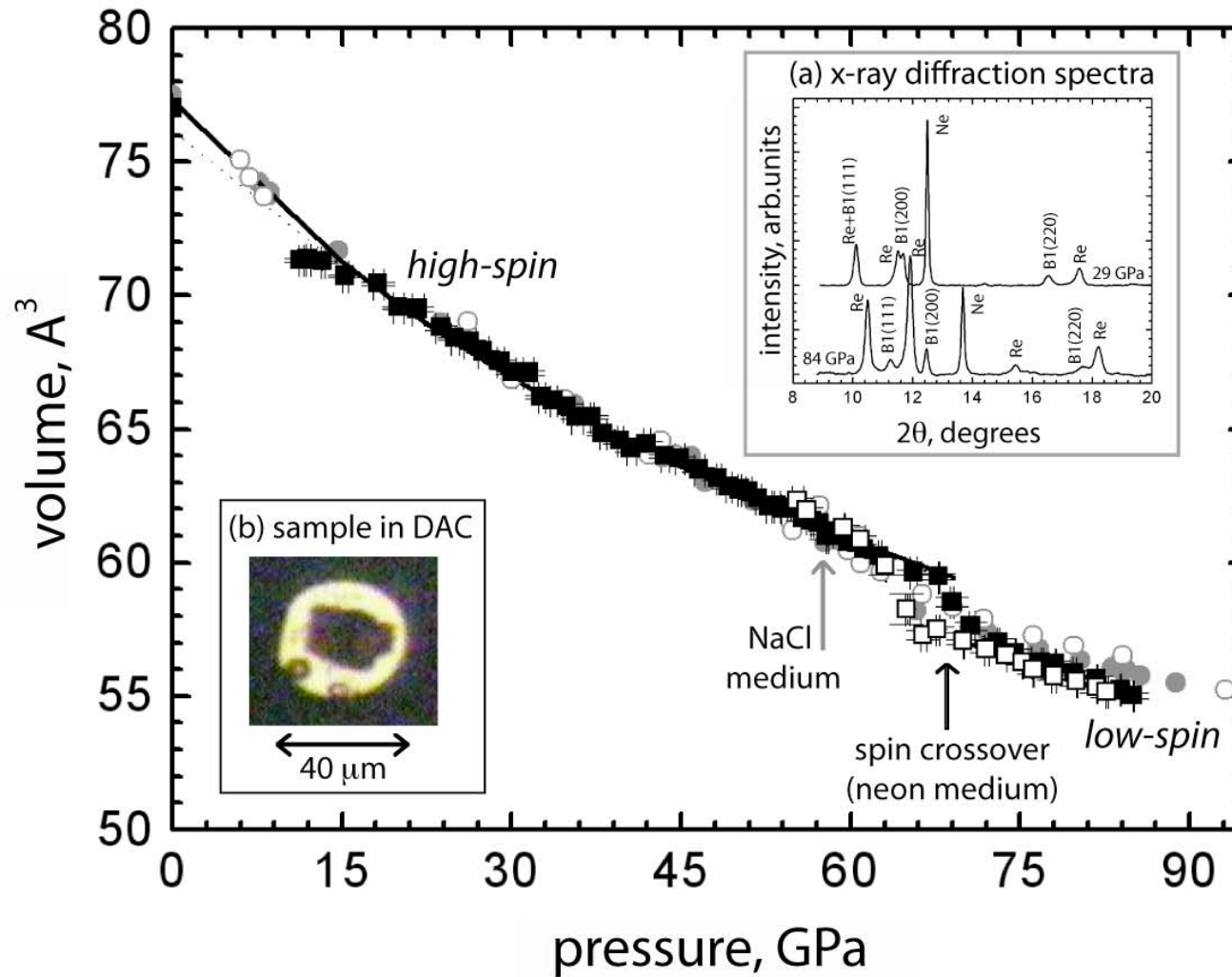
- Determine d-spacings for (mostly) known structures as a function of pressure



$$\lambda = 2d \sin \theta$$

Volume (density) change of $(\text{Mg}_{0.61}\text{Fe}_{0.39})\text{O}$ through the spin crossover

Experiments conducted at The Advanced Light Source, Berkeley, CA



Zhuravlev, Jackson, Wolf, Wicks, Yan, Clark (2010)
Phys. Chem. Min.

Other X-ray Diffraction Methods

- Small angle x-ray scattering
 - Structural deviations
 - Local scale
- Single-crystal x-ray diffraction
 - Can be done in-house or at the synchrotron
 - Solves for atomic positions
 - Solves structure
 - Structures of minerals
 - Protein crystallography

Our XRD experiments

339 Steele

- Sample preparation
 - Grind under alcohol using agate mortar & pestle
 - Grain size should be $\sim 100 \mu\text{m}$
 - Can influence quality of XRD spectrum
- Copper target
- Ni filter
- $\text{Cu K}\alpha_1$, $\lambda = 1.54056 \text{ \AA}$
- $\text{Cu K}\alpha_2$, $\lambda = 1.54439 \text{ \AA}$
- 8 keV
- Bragg's law:
$$n\lambda = 2d\sin\theta$$

